User policies in a network implementing congestion pricing

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Abstract

In this paper we consider a network implementing congestion pricing. For general user utility functions, and in a regime where user peak rates are small compared to network link capacities, we establish optimality of the considered pricing scheme. The corresponding optimal user policies are perhaps contrary to what one would expect: file transfers must either be done at maximal speed, or have access denied, whereas real time applications will display elasticity in their choice of sending rates. We also discuss how the optimality property is affected by price encoding mechanisms implemented in the network, and the resulting effect on user policies.

1 Introduction

The problem of quality of service provision in the Internet has attracted a great deal of attention. Integrated Services, the first approach proposed by the Internet Engineering Task Force (IETF, see [3]), relied on the reservation protocol RSVP, which enabled users to make bandwidth and buffer reservations along a path, upon connection. However, questions have been raised about its scalability, since routers have to maintain per connection states.

IETF attention has recently turned towards the provision of "Differentiated Services" (DiffServ), an approach which is no longer based on per flow management in the network, but rather identifies a number of Quality of Service classes. It asks the network to make a per class assignment rather than per flow. The scalability issue vanishes because of the small number of classes. Although commercial Cisco routers supporting DiffServ are already available, the corresponding ideal network management is far from obvious, and neither is the choice of adequate tariffs for each class.

Yet a third approach has been proposed as a challenger to DiffServ [4, 5, 6]. It assumes a single class in the network, and only requires routers to tag packets, a tag corresponding to the spot price of forwarding a packet at the precise instant under consideration. End users are then informed of the price incurred when acknowledgements flow back to them, and are then free to decide when or whether to send additional packets. This not only solves the pricing issue facing current DiffServ proposals, but also appears to be the most flexible approach by allowing end users to design their own strategies in view of their personal needs.

The aim of this note is to assess this scheme's optimality, and review associated implementation issues and possible solutions. We consider a network handling both file transfers and real-time connections with associated general utility functions. Assuming spot prices can take an infinite number of values, we show that, when the link's bandwidths are large compared to end users peak rates, the system reaches

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an equilibrium which achieves near-maximal welfare. Interesting user strategies emerge: in particular, file transfers either occur at peak rate, or are abandoned. This is contrary to the current intuition that file transfer is an elastic application, which should not undergo admission control, but instead adapt its rate to the network's status (note however, that management of file transfer applications via admission control has already been advocated in [8, 10], as a means of improving efficiency).

We then address how this near-optimality property is affected when spot prices can take only a limited number of values. This leads us to consider price assessing strategies and risk evaluations by the end users. We show a generalization to TCP's slow-start behaviour arises as a natural control scheme in such a context.

2 A simplified fluid model

Consider a network that consists of a number of links l with capacities C_l. The connections supported by the network in what follows are point-to-point; each connection proceeds along some route identified with a sequence of network links.

File transfers can be of several types, indexed by $i \in I$. Type i files have size V_i, can be sent at a rate not larger than π_i, and the utility of transfering them in T seconds is $U_i(T)$, where U_i is some function decreasing to zero as T increases. Hence, the utility of a file transfer only depends on its completion time, and is decreasing in this completion time. Requests for file transfers of type i occur at the instants of a Poisson process with intensity Λ_i. Finally, file transfers of a given type i proceed along a fixed route; we write $l \in i$ if link l belongs to that route.

Real time connections can be of several types, indexed by $j \in J$. Type j real-time connections have duration T_j, have a peak rate π_j, and the utility of allocating them time-varying rate $x(t)$ equals $\int_0^{T_j} U_j(x(t))dt$ for some non-decreasing function U_j such that $U_j(0) = 0$, and admitting the general shape displayed on figure 1, which is discussed further in the next section. Requests for type j connections occur at the instants of a Poisson process with intensity Λ_j; they proceed along a fixed route; again we write $l \in j$ if link l belongs to the corresponding route.

If at time t, link l is faced with an instantaneous arrival rate of x packets per second, it will charge all forwarded packets a fee of $f_l(x/C_l)$, for some non-decreasing continuous function f_l. Hence, a user contributing some proportion θ to rate x incurs an instantaneous cost rate at link l of $\theta x f_l(x/C_l)$. We assume in the next session that users are always aware of the price they currently incur; a more realistic set-up is given in Section 4, with the necessary adjustments.

3 The many small users regime: fixed-point equations and near-optimality

In the case where users have perfect knowledge of the future behaviour of price, and of their own actions' impact on price, they can correctly assess what policy they should use to maximise their personal gain, and in particular whether this gain can be positive so that it is worth connecting or not. If, for instance, users are confident that the spot price per packet along their route – which is the sum of the spot prices at each link of the route – is going to be constant throughout the transfer, and not influenced by their

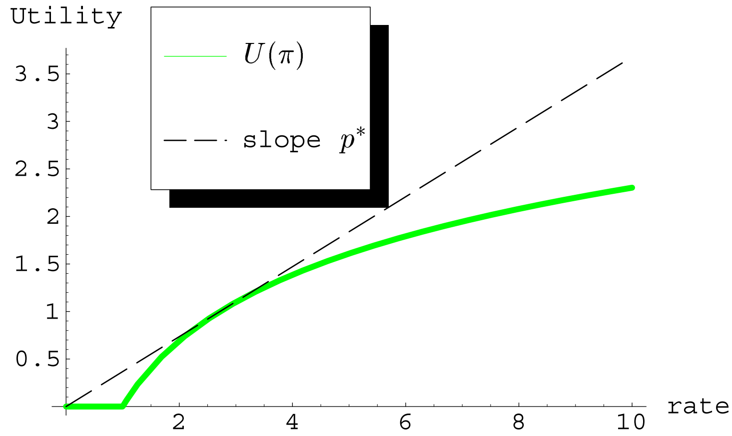

Figure 1: Utility of rate to real time users.

own policy, the following strategies will emerge. For a type i file transfer, the utility of transfering at some rate $\pi \in (0, \pi_i]$ is given by

$$U_i(V_i/\pi) - V_i p$$

where p is the corresponding spot price. Hence, it is optimal to take $\pi = \pi_i$, and the associated gain will be non-negative provided $p \leq p_i^*$, where

$$p_i^* = U_i(V_i/\pi_i)/V_i.$$

In other words, it is beneficial for the user to connect if and only if $p \leq p_i^*$, in which case gain is maximised by sending data at peak rate.

Consider now real-time transfers. We assume the function U_j is such that $U_j(0) = 0$, is upper-bounded on the interval $[0, \sigma_j]$ by a straight line with slope p_j^* and is tangent to that line at σ_j, after which it is strictly concave (see figure 1). For a type j real-time transfer, it will be optimal to transfer at rate $\pi \in (0, \pi_j]$ which maximises $U_j(\pi) - \pi p$. Provided $p < p_j^*$, this maximum is attained at a single value $\gamma_j(p) := U_j'^{-1}(p)$, and $\gamma_j(p)$ is decreasing in p. If $p > p_j^*$, this maximum is realized at $\gamma_j(p) = 0$. The associated gain will always be non-negative so that it is always beneficial for customers to enter the system. We shall also make use of $\bar{U}_j$, which is the smallest concave function majorizing U_j.

We now consider a scaling regime under which prices do indeed tend to stabilize, and individual users' impact on price does indeed vanish. With N a large parameter, the scaling assumptions are as follows: link capacity C_l equals Nc_l, arrival rates Λ_i, Λ_j equal $N\lambda_i$ and $N\lambda_j$, respectively where the parameters c_l, λ_i, λ_j are fixed. Theorem 1 below illustrates how, when users behave as described above, and in this scaling regime, the whole system converges to some optimal operating point. Before stating it, we provide fixed-point equations which can be used to determine these optimal operating points. Recall that f_l determines the spot price of forwarded packets as a function of the load at link l. Let γ_k represent the equilibrium sending rate (rescaled by N) due to type k users, where $k \in I \cup J$, and p_l the equilibrium spot price at link l. Then these quantities should satisfy

$$p_l = f_l\left(\left[\sum_{k \in I \cup J : l \in k} \gamma_k\right]/c_l\right),\ l \in L \tag{1}$$

$$\lambda_i V_i \mathbf{1}_{p_i < p_i^*} \leq \gamma_i \leq \lambda_i V_i \mathbf{1}_{p_i \leq p_i^*},\ i \in I \tag{2}$$

$$\lambda_j T_j \gamma_j(p_j) \mathbf{1}_{p_j < p_j^*} \leq \gamma_j \leq \lambda_j T_j \gamma_j(p_j) \mathbf{1}_{p_j \leq p_j^*},\ j \in J, \tag{3}$$

where $p_k = \sum_{l \in k} p_l$ for all $k \in I \cup J$. In general there may exist several solutions to these equations.

We now introduce some more notation. We shall assume first that type i file transfers connect upon arrival if and only if the current spot price is less than their critical price, after which they send at maximal rate π_i, and that real time transfers always connect, and remain connected for the whole duration T_j, while sending at rate $\gamma_j(p)$, where p is the spot price they detected upon connection. Note that when $p > p_j^*$, this is equivalent to not connecting, since in that case $\gamma_j(p) = 0$. These user policies are consistent with the above discussion. We then have the following

Theorem 1 . *Let x_i denote the number of ongoing type i file transfers rescaled by a factor N. Let x_j denote the number of ongoing real time transfers, rescaled by a factor N too. When N goes to infinity, the limiting quantities evolve towards a point maximizing the objective function*

$$\sum_i p_i^* \pi_i x_i + \sum_j x_j \bar{U}_j(\gamma_j) - \sum_l \int_0^{\sum_{i \ni l} \pi_i x_i + \sum_{j \ni l} x_j \gamma_j} f_l\left(u/c_l\right) du \tag{4}$$

over $\gamma_j \geq 0$, $x_i \in [0, \lambda_i V_i/\pi_i]$, $x_j \in [0, \lambda_j T_j]$. Denote by p_i, p_j the attained equilibrium prices. Provided they all differ from the corresponding critical prices p_i^*, p_j^*, the following identities hold:

$$\begin{cases} x_i = \lambda_i V_i / \pi_i \mathbf{1}_{p_i^* < p_i}, \\ x_j = \lambda_j T_j, \\ \gamma_j = \gamma_j(p_j). \end{cases} \tag{5}$$

Remark 1 . *Convergence of the limiting rescaled process to a point maximizing the objective function (4) does in fact hold in a more general set-up. It still holds, for instance, if the real time users, instead of maintaining a fixed sending rate through their connection, adapt their rate towards the current best value $\gamma_j(p_j)$.*

The efficiency of equilibrium point (5) depends upon properties of the functions f_l. Assume $f_l(z) = 0$ for $z \leq \alpha$, and $f_l(z) > \sup(p_i^*, p_j^*)$ for $z \geq \beta$, for constants α, β satisfying $0 < \alpha < \beta < 1$. Then this equilibrium achieves greater welfare than what could be achieved in the same network but where the capacity limits of the links are αC_l instead of C_l; on the other hand, it ensures that capacity limits of βC_l are not exceeded. When α is close to 1, achieved welfare is in turn close to maximal welfare, while having β less than 1 ensures that capacity limits are indeed respected. The proof of the theorem relies on a classical Lyapunov function technique, used for instance in Kelly [5], where in contrast to the modelling in the present note, users are viewed at a higher level of aggregation. Related models and analyses can be found in Paschalidis and Tsitsiklis [9], and Tan [11].

In order to illustrate the previous result, consider the case of a single link l. Assume there is only one type of file transfer, say type 1, and only one type of real time application, say type 2. The equation characterizing the equilibrium price is then

$$p = f\left(\frac{1}{c}\left[\lambda_1 V_1 \mathbf{1}_{p \leq p_1^*} + \lambda_2 T_2 \gamma_2(p) \mathbf{1}_{p \leq p_2^*}\right]\right),$$

where the subscript l has been dropped. To make this more explicit, assume that $U_2(x) = \max(w \log(x), 0)$ for some constant $w > 0$. In that case, $p_2^* = we^{-1}$, and $\gamma_2(p) = w/p$. We now illustrate how function

f might arise from some marking scheme based on the packet level behaviour of the system (a detailed discussion of such marking schemes can be found in [7]). Suppose packets are charged 0 or 1 (in cents, say) according to whether they are queued behind less than, or at least b packets at the link's access respectively. Assume packet dynamics are fast compared to the dynamics at connection level, and packet arrivals occur according to a Poisson process with rate as in the fluid model. Assuming independent identically distributed packet sizes, with exponential distribution of parameter 1, a plausible choice is

$$f(x) = \min(x^b, 1).$$

Indeed, this corresponds to the stationary marking probability in the M/M/1/∞ queue with load x. Note that this function would represent the current *average* spot price, the exact spot price oscillating quickly between the two values 0 and 1.

4 Towards practical user policies

The user policies considered in the previous section originate from users being aware of the current spot price, and being confident that this price will be stable in time. Such stability of prices in time, although holding at equilibrium in the many small users regime, when one assumes a continuum of price values can be conveyed at each time to the user, might not hold in other regimes of interest.

Consider again the threshold marking scheme of the previous session, with price signals alternating between 0 and 1. Users should no longer assume prices are constant, but if they are confident in price stability they might now assume price signals are independent and identically distributed. A sensible approach (see [7]) consists in sending a number M of probing packets, and deciding whether or not to join depending on the number of marked packets received. In a Bayesian paradigm, one can assume a prior probability distribution on the probability p of receiving a mark for any packet. For instance, consider a Beta distribution for p with parameters α, β, i.e. the probability density of p is proportional to $p^{\alpha-1}(1-p)^{\beta-1}$. Then, conditional on obtaining k marks out of M probe packets, the posterior probability distribution on p is Beta, with parameters $(\alpha+k, \beta+M-k)$. The corresponding average value $\bar{p}$ equals

$$\frac{\alpha+k}{\alpha+\beta+M}.$$

Consider, for instance, a file transfer application with associated critical price p^*. A natural policy for users of the corresponding type consists then in deciding to join provided $\bar{p} \leq p^*$, and not joining otherwise. That is to say, users join if and only if

$$k \leq k_0 := p^*(\alpha+\beta+M) - \alpha.$$

Indeed, such a policy maximises the user's expected utility, which is the desirable objective of *risk neutral* users. Once such thresholds are determined for each class, it is again possible to write fixed point equations for the equilibrium prices. Consider for simplicity a single link, supporting a single class of file transfers, say class 1. Then the true value of the price p will satisfy

$$p = f\left(\lambda_1 V_1 \sum_{k=0}^{k_0} \binom{M}{k} p^k (1-p)^{M-k}\right).$$

Such fixed point equations allow us to determine the efficiency in terms of conveyed welfare, as compared to the fixed point equations derived in the previous section.

Risk averse users will typically put a stronger weight against negative utility. We now discuss a plausible policy for such users. Again, assume users have an a-priori distribution on the marking probability which we now take to be Beta(1,1), i.e. a uniform distribution on $[0,1]$, and evaluate their risk at any time by the following quantity

$$R_\theta := \mathbf{E}\left(e^{\theta \sum_1^n (p_i - p^*)}\right),$$

where $\theta > 0$ is some parameter quantifying their averseness to risk, n is the total number of packets sent and not yet acknowledged, p^* is the target price per packet, p_i is the effective price of the ith packet, and $\mathbf{E}$ denotes expectation given the information currently available. Assume that users will always act so as to maintain R_θ below some fixed threshold.

If the user has already received the acknowledgments for m packets sent, out of which a proportion q has been marked, one has the following expression:

$$R_\theta = e^{-\theta n p^*} \frac{\Gamma(m)}{\Gamma(mq)\Gamma(m(1-q))} \int_0^1 p^{mq}(1-p)^{m(1-q)} \left(pe^\theta + 1 - p\right)^n dp.$$

A simple approach to an approximate formula consists in assuming m and n are large, with fixed ratio $n/m = \alpha$, and applying Laplace's method to get a logarithmic equivalent of the integral, of the form

$$\log R_\theta \sim m F(q, p^*, \alpha).$$

Note that αm is exactly the number of unacknowledged packets, or the window size in a window based flow control. A ratio α will be deemed feasible if the exponent $F(q, p^*, \alpha)$ in the previous evaluation is negative, for otherwise the risk is uncontrolled. This thus defines a critical ratio $\alpha(q, p^*)$, which is the largest root of

$$F(q, p^*, \alpha) = 0.$$

This discussion thus motivates the choice of the window size as $m\alpha(q, p^*)$, where m is the number of acknowledgements received, q is the measured marking probability, and p^* is the upper bound on marking probability. Note that, assuming measured values of q are stable, this amounts to increasing the window size by the fixed amount $\alpha(q, p^*)$ upon each acknowledgement receipt, which generalizes TCP's slow start behaviour, where such an increment is chosen to have unit size.

We now make this more precise, with evaluations of functions F and α. First, $F(q, p^*, \alpha)$ admits the expression:

$$F(q, p^*, \alpha) = -q\log(q) - (1-q)\log(1-q) - \alpha\theta p^* + \sup_{p \in (0,1)} \left(q\log(p) + (1-q)\log(1-p) + \alpha\log(e^\theta p + 1 - p)\right).$$

In Figure 2 below, F is plotted against α for $q = 0.1$, $e^\theta = 2$, each curve corresponding to one choice of p^* in $\{0.15, 0.2, 0.25, 0.3\}$. A close examination of the curves shows that the corresponding values for $\alpha(q, p^*)$ are given by 0.2, 0.83, 1.3 and 1.76 respectively.

We next investigate the sensitivity of these quantities with respect to θ. Figure 3 displays the curves $F(q, p^*, \alpha)$ as a function of α, for $q = .1$, $p^* = .3$, and e^θ taking values in $\{1.5, 2, 2.5, 3\}$. From these curves one obtains that the corresponding values of $\alpha(q, p^*)$ are given by 3.6, 1.76, 1.14 and 0.83 respectively.

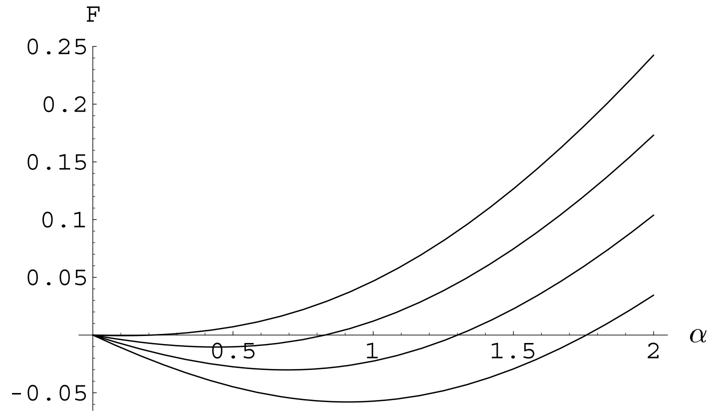

Figure 2: F as a function of α, for $q = .1, e^\theta = 2$, $p^* \in \{0.15, 0.2, 0.25, 0.3\}$.

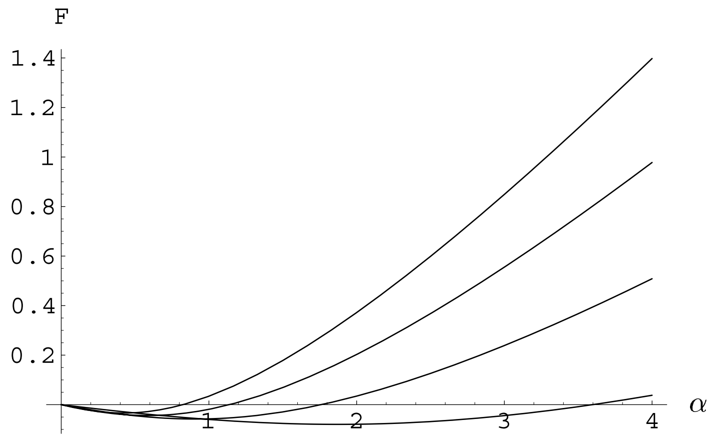

Figure 3: F as a function of α, for $q = .1$, $p^* = .3$ and $e^\theta \in \{1.5, 2, 2.5, 3\}$.

For example, a user with $e^\theta = 1.5$, faced with an empirical marking rate of 0.1 and attributing a value of 0.3 to each packet should, according to this analysis, increase the window size by 3.6 upon arrival of each acknowledgement. Note that the current estimate of q has to be updated at each reception of an acknowledgement; however, it should eventually converge to the true value of the marking probability.

5 Conclusions

In this note we have investigated likely user policies for both file transfer and real time elastic applications in a network implementing congestion pricing. Fixed point equations characterizing equilibrium behaviour are derived in both cases where price information can take values in a continous or in a finite set. Near-optimality of the scheme is proven in the case of continuum-valued prices. File transfer applications are typically expected either to join at access speed ar not to join. This feature stems from users being confident in price stability, and unaware of any impact of their behaviour on the price. We have also discussed plausible behaviour of risk-averse users who want to assess the true price of packets, where a generalization of TCP's slow start mechanism emerges as a natural policy. The results presented here apply to situations where variations of the demand put on the network are small. It is a challenging issue to understand how the conclusions are affected when demand varies more significantly.

References

[1] P. Barham, P. Key, K. Laevens and D. McAuley, Congestion pricing for congestion avoidance, Microsoft Research Technical Report MSR-TR-99-15, MSR, 1999.

[2] S. Floyd and K. Fall, Router mechanisms to support end-to-end congestion control. Lawrence Berkeley National Laboratory, Preprint (1997).

[3] The use of RSVP with IETF Integrated Services, IETF RFC 2210, available at http://www.ietf.org/html.charters/intserv-charter.html.

[4] R. J. Gibbens and F. P. Kelly, Resource pricing and the evolution of congestion control, Automatica (1999) 35, pp. 1969-1985.

[5] F. Kelly, Charging and rate control for elastic traffic. Europ. Trans. Telecom. Vol 8 (1997), 33-37.

[6] F. Kelly, A. Maulloo and D. Tan, Rate control for communication networks: shadow prices, proportional fairness and stability. Journal of the Operational Research Society 49 (1998).

[7] F. P. Kelly, P. B. Key and S. Zachary, Distributed admission control, IEEE JSAC, Vol. 18 (2), pp. 2617-2628 (2000).

[8] L. Massoulié and Jim Roberts, Advocating admission control for TCP flows, ITC 16, Edinburgh (1999).

[9] Y. Paschalidis, J. Tsitsiklis, Congestion-dependent pricing of network services, IEEE Transactions on Networking, Vol. 8 (2), pp. 171-184 (2000).

[10] J. Roberts and L. Massoulié, Bandwidth sharing and admission control for elastic traffic, ITC Seminar, Yokohama (1998).

[11] D. Tan, Mathematical models of rate control for communication networks, Ph. D. thesis (1999).